DISPEC Training School

Ionosphere Physics Fundamentals

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Outline

1. **Signal Propagation**
2. **Ionosphere Structure**
3. **Ionospheric Delay in GNSS Signals**
4. **Ionospheric Sounding with GNSS Measurements.**

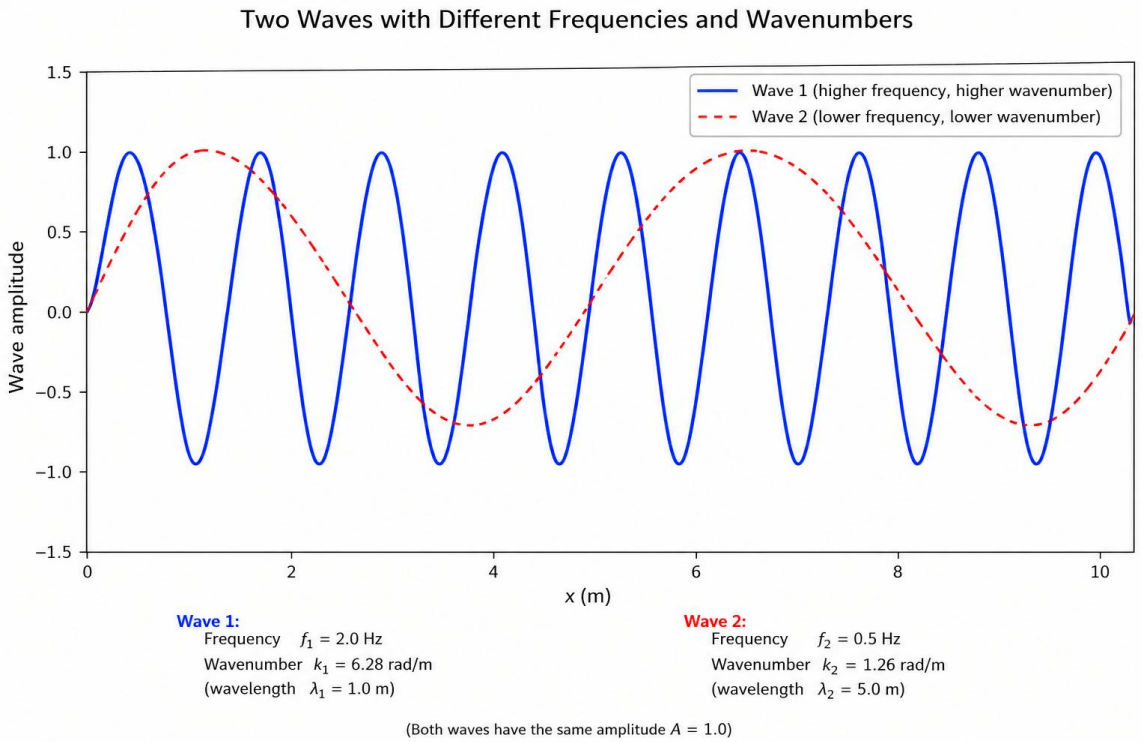
Ionosphere Physics. Signal Propagation

Group and Phase velocities

Let's consider two different electromagnetic plane waves in the propagation direction and their superposition:

$$\begin{cases} \psi_1 = \cos(\omega t - kx) \\ \psi_2 = \cos[(\omega + \delta\omega)t - (k + \delta k)x] \end{cases}$$

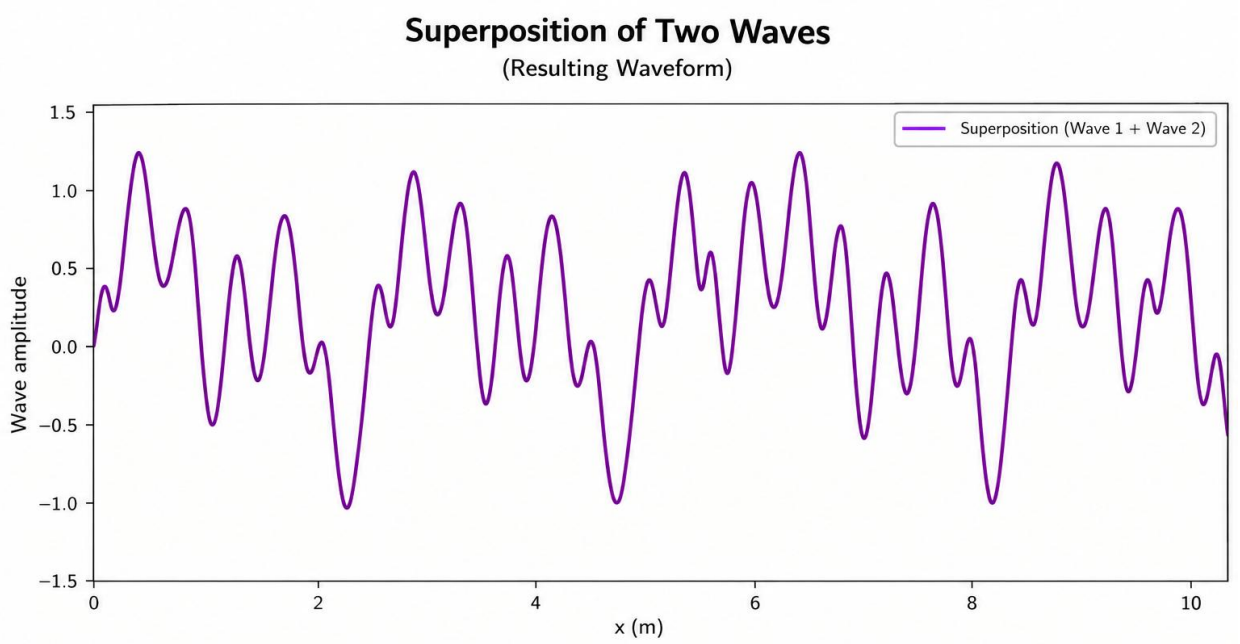
$$\psi = \psi_1 + \psi_2 \simeq \cos\left[\frac{1}{2}(\delta\omega\, t - \delta k\, x)\right] \cdot \cos(\omega t - kx)$$



Wave 1:
Frequency $f_1 = 2.0$ Hz
Wavenumber $k_1 = 6.28$ rad/m
(wavelength $\lambda_1 = 1.0$ m)

Wave 2:
Frequency $f_2 = 0.5$ Hz
Wavenumber $k_2 = 1.26$ rad/m
(wavelength $\lambda_2 = 5.0$ m)

(Both waves have the same amplitude $A = 1.0$)



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Resulting Wave (Superposition):
($f_1 + f_2$ components)
Not a single frequency or wavenumber
(it contains both)

(The superposition produces a wave with a complex pattern due to the different frequencies and wavenumbers.)

Group and Phase velocities

In the resulting superposition, we have the **phase velocity** v_p for the carrier phase and the **group velocity** v_g, the velocity of the transmitted information :

$$\begin{cases} v_p = \dfrac{\omega}{k} \\ v_g = \dfrac{d\omega}{dk} \end{cases} \quad \Longrightarrow \quad v_g = v_p + k\,\frac{dv_p}{dk}$$

Then, the refraction indices, i.e. the ratio of the speed of light and phase/group speed,

$$n_g = \frac{c}{v_g}, \qquad n_p = \frac{c}{v_p},$$

are related as follows:

$$\boxed{n_g = n_p + f\,\frac{dn_p}{df}}$$

Ionosphere Physics. Signal Propagation

Electromagnetic Waves

The solution of the Maxwell equations in the vacuum (ρ=0) is the electromagnetic planar waves with the phase velocity constrained by the vacuum permittivity and permeability, ε_0 and μ_0, respectively;

$$\nabla \cdot \vec{D} = \rho, \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t},$$

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

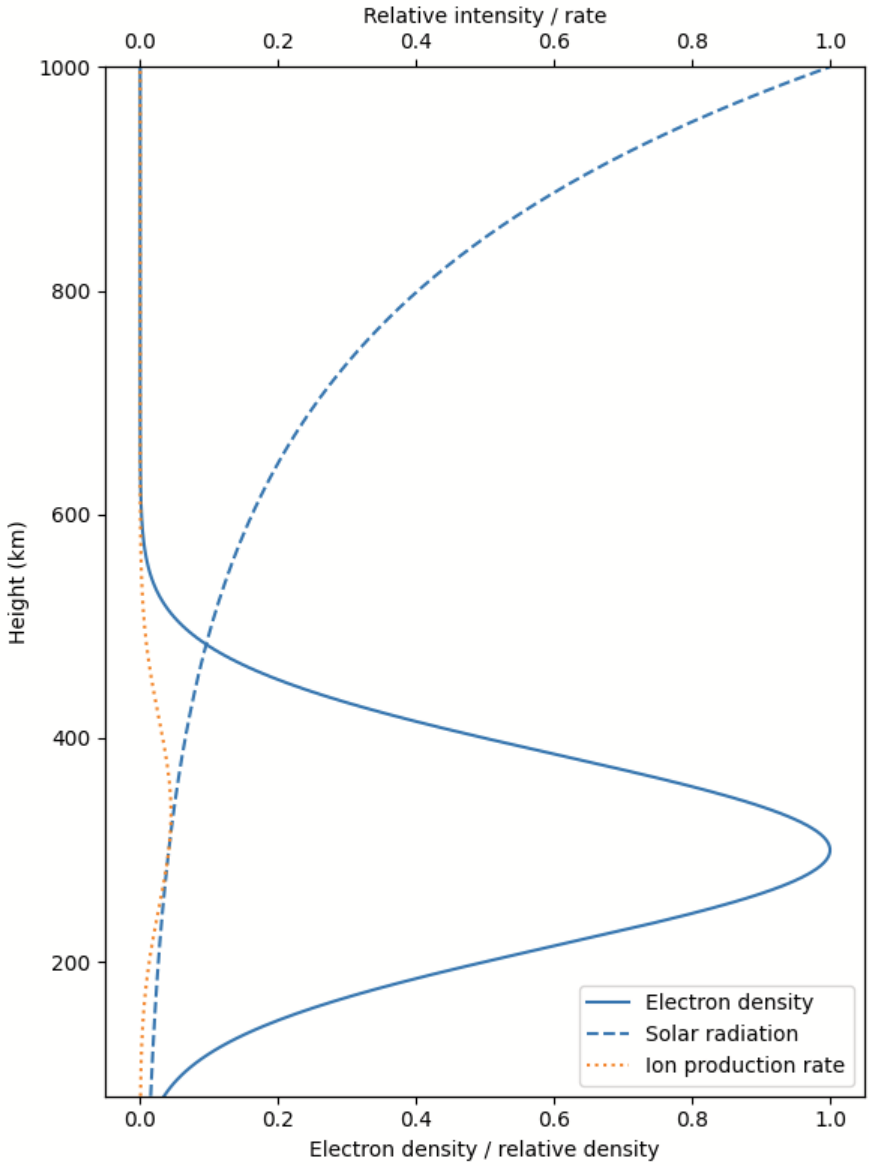
$$\begin{cases} \nabla^2 \vec{E} = \varepsilon_0 \mu_0 \dfrac{\partial^2 \vec{E}}{\partial t^2}, \\ \nabla^2 \vec{H} = \varepsilon_0 \mu_0 \dfrac{\partial^2 \vec{H}}{\partial t^2}, \end{cases} \quad \Longrightarrow \quad v_p = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c.$$

Ionosphere Physics. Ionosphere Structure

Distribution of free electrons and ions in the atmosphere

The distribution of electrons and ions depends on:

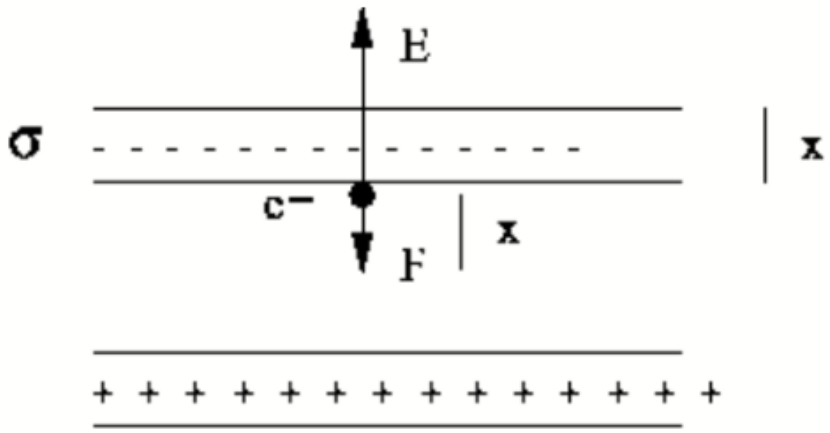
- Intensity of the solar radiation (ISR), which increases with height because the atmospheric absorption decreases with height.
- Density of the ionized gas (Dig), which decreases with height, as the atmospheric density does as well.
- Ion production rate (IPR), which results from the combination of the above conditions.



Ionosphere Physics. Ionosphere Structure

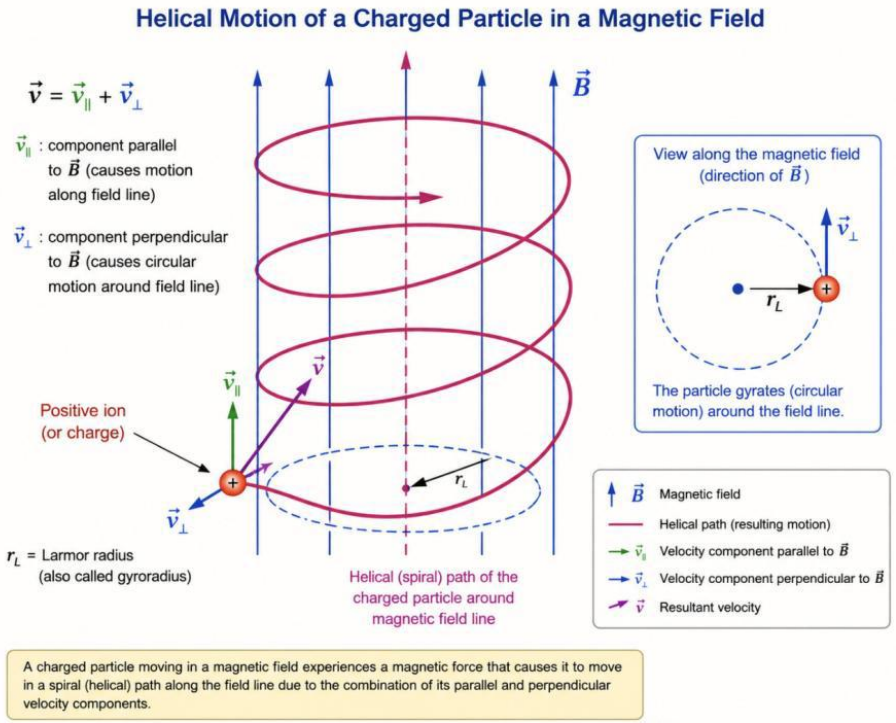
Distribution of free electrons and ions in the atmosphere

Oscillations of the electrons in the neighbourhood of the ions.



$$\omega_N = \sqrt{\frac{Ne^2}{m\varepsilon_0}}, \qquad \omega_N^i = \sqrt{\frac{Ne^2}{M\varepsilon_0}}$$

Movement of electrons and ions around the magnetic field lines in a spiral path.



$$\omega_H = \frac{eB}{m} \qquad \omega_H^i = \frac{m}{M}\omega_H$$

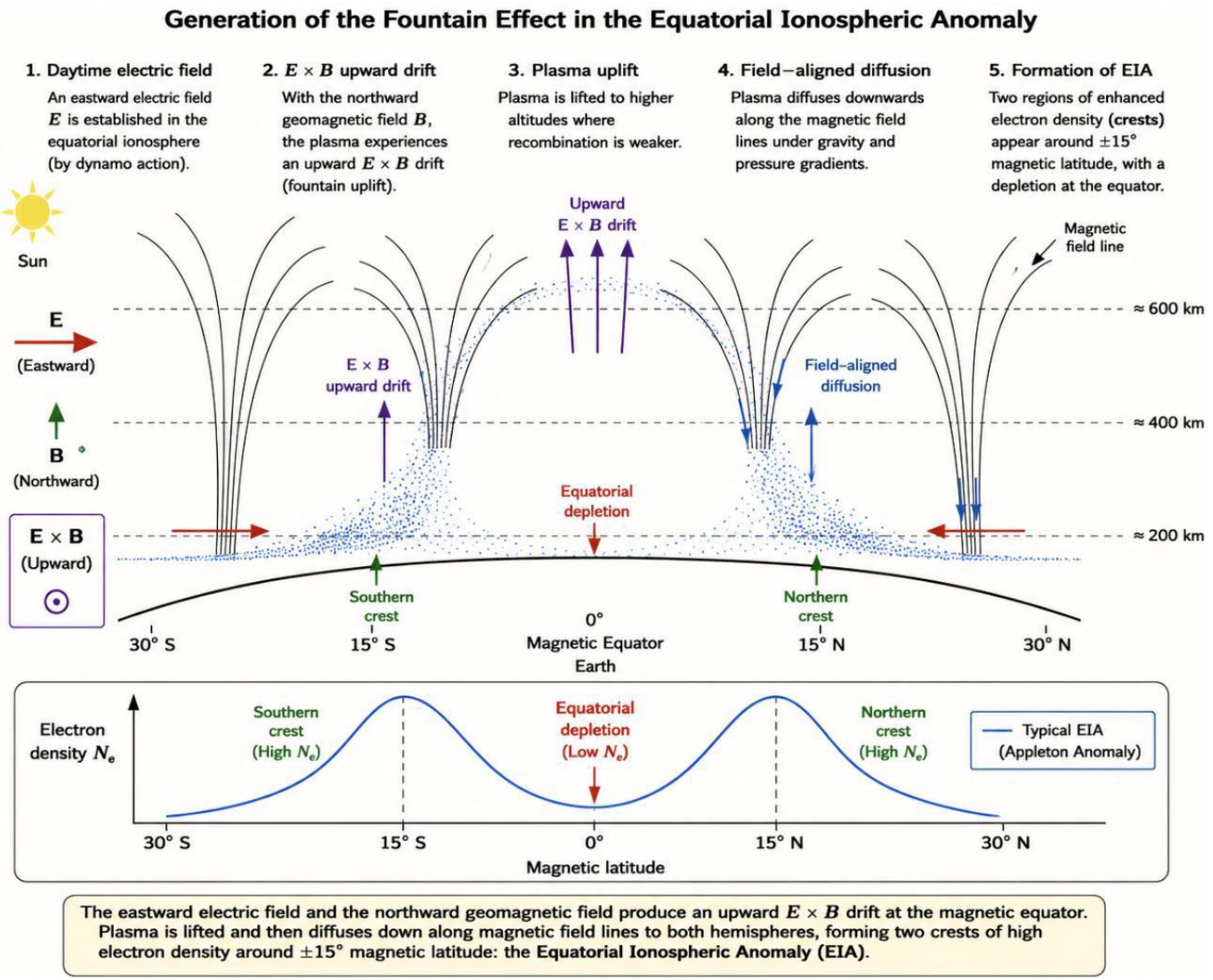
Ionosphere Physics. Ionosphere Structure

Distribution of free electrons and ions in the atmosphere

The eastward winds in the dayside region of the atmosphere drag ions and free electrons differently, thus generating an electric field **E** in the same direction.

The combination of the eastward E and the northward geomagnetic field B produces an upward drift velocity (Lorentz Force)

$$\vec{v}_{drift} = \frac{\vec{E} \times \vec{B}}{B^2}$$



Ionosphere Physics. Ionospheric Delay

Ionospheric Delay on GNSS signals. Phase and Code

Let's consider the effect of free electrons on an EM signal (e.g. from a GNSS transmitter).

- In the vacuum, the phase velocity is $v_p = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \mathrm{c}$, and $n_p = 1$
- In the presence of free electrons:

$$\varepsilon_{\mathrm{ef}} = \varepsilon \left[1 - \frac{e^2 n_e}{m\omega^2 \varepsilon_0} \right]$$

$$n_p = \frac{c}{v_p} = \cdots = 1 - \frac{e^2 n_e}{m\omega^2 \varepsilon_0} \simeq 1 - \frac{40.3\, n_e}{f^2}$$

- Then, the refractive index of the group depends on the electron density and the frequency of the EM signal as follows:

$$n_g = 1 + \frac{40.3\, n_e}{f^2}$$

Ionosphere Physics. Ionospheric Delay

Ionospheric Delay on GNSS signals. Phase and Code

The ionospheric delay is the difference between the paths travelled by an EM wave throughout a plasma fuild and vacuum, i.e.

$$\Delta s_g = c \int_{\vec{r}_j}^{\vec{r}} \frac{dS}{v_g} - \int_{\vec{r}_j}^{\vec{r}} dS = \int_{\vec{r}_j}^{\vec{r}} (n_g - 1)\, dS$$

$$\Delta s_g = -\Delta s_p = \frac{40.3\, I}{f^2}$$

Slant Total Electron Content (**STEC**)

$$I = \int_{\vec{r}_j}^{\vec{r}} n_e\, dS$$

This term accounts for 99.9% of the total ionospheric delay. For further details on **higher-order terms**, see Hernández-Pajares, M., À. Aragón-Ángel, P. Defraigne, N. Bergeot, R. Prieto-Cerdeira, and A. García-Rigo (2014), Distribution and mitigation of higher-order ionospheric effects on precise GNSS processing, J. Geophys. Res. Solid Earth, 119, 3823–3837, doi:10.1002/2013JB010568.

1 TECU = 10^{16} electrons/m^2

Ionosphere Physics. Ionospheric Delay

Ionospheric Delay on GNSS signals. Phase and Code

$$P_1 = \rho^j + c(dt - dT) + \frac{40.3\,I}{f_1^2} + \cdots$$

$$P_2 = \rho^j + c(dt - dT) + \frac{40.3\,I}{f_2^2} + \cdots$$

$$L_1 = \rho^j + c(dt - dT) - \frac{40.3\,I}{f_1^2} + \cdots$$

$$L_2 = \rho^j + c(dt - dT) - \frac{40.3\,I}{f_2^2} + \cdots$$

P_1 and **P_2** are the measurements of P-code, or pseudodistance.

- **L_1** and **L_2** are the measurements of the carrier-phase.
- ρ_j is the geometric distance to satellite j.
- dt and dT are the clock errors for the receiver and satellite, respectively.
- I is the **STEC**
- … other terms

Note that the ionospheric delay I has a negative sign in the carrier-phase equations.
To ***summarize***, the effect of the ionosphere on the GNSS measurements is two-fold:

- It yields a delay on the code.
- It reduces the carrier-phase path.

Ionosphere Physics. Ionospheric Sounding

Ionospheric Sounding with GNSS measurements

$$P_1 = \rho^j + c(dt - dT) + \frac{40.3\,I}{f^2} + \cdots$$
$$P_2 = \rho^j + c(dt - dT) + \frac{40.3\,I}{f_2^2} + \cdots$$
$$L_1 = \rho^j + c(dt - dT) - \frac{40.3\,I}{f_1^2} + \cdots$$
$$L_2 = \rho^j + c(dt - dT) - \frac{40.3\,I}{f_2^2} + \cdots$$

Ionospheric combination of GNSS measurements LI and PI

$$LI \equiv L_1 - L_2 = 40.3 \cdot \left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) \cdot STEC + B_1 - B_2$$

$$PI \equiv P_2 - P_1 = 40.3 \cdot \left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) \cdot STEC + DCB - DCB^j$$

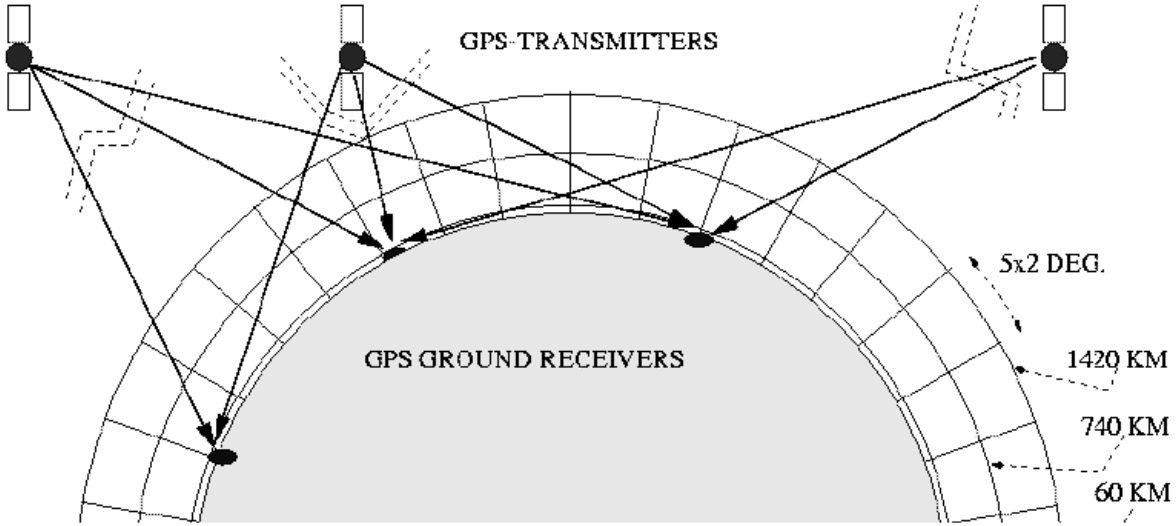
B_1 and **B_2**: ambiguity terms from the integration of the Doppler signal to estimate the carrier-pase signal.

DCBs are the Delay Code Biases, which is the result of different timing operations of the hardware in both the receiver and the transmitter.

It is possible to estimate the STEC, therefore the electron content, alongside with ambiguities and DCBs.

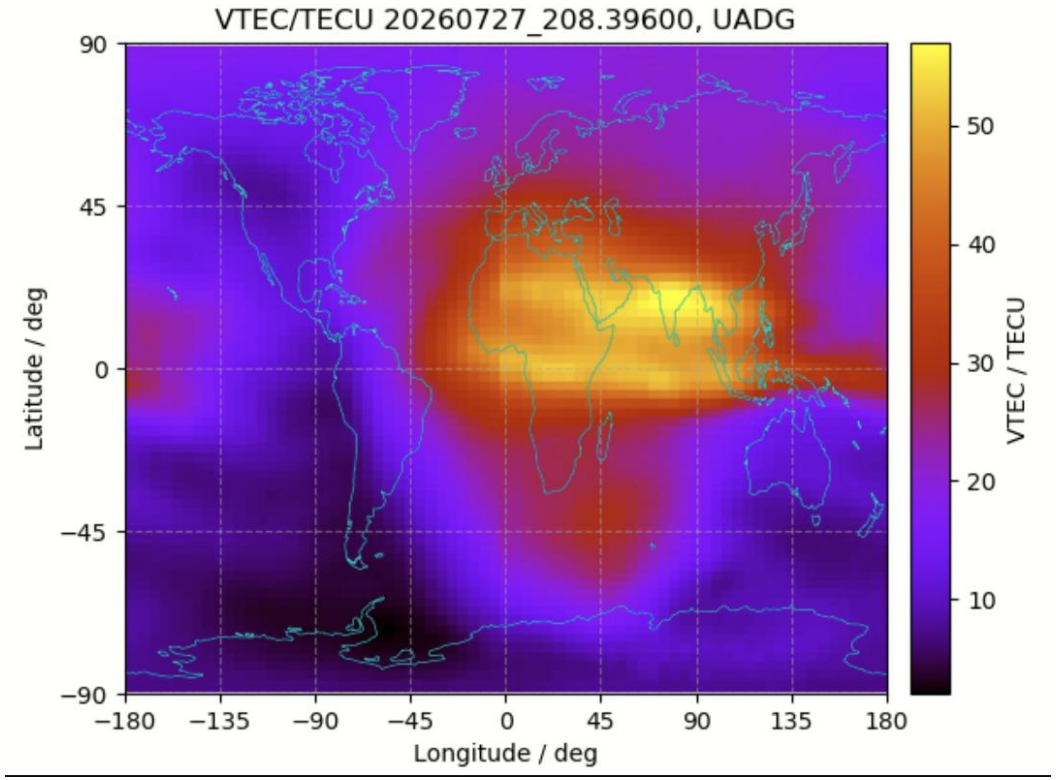
Ionosphere Physics. Ionospheric Sounding

Tomographic Ionospheric Sounding with GNSS measurements



$$STEC = \int_r^t N_e \, dl \approx \sum_{\#voxels} N_e \delta l$$

Global Ionospheric Maps from GNSS tomograhy



Thank you for your attention!

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